Mathematical Modelling for TEWL Calibration Using Droplet Method

P Xiao[1,2], M E P de Jesus[3], Y Cui[1], R E Imhof[1,2] and the TEWL Calibration Consortium[4]

1 Photophysics Research Centre, London South Bank University, London SE1 0AA, UK

2 Biox Systems Ltd, Southwark Campus, 103 Borough Road, London SE1 0AA, UK

3 Departamento de Fisica Universidade da Beira Interior, 6200 Covilhã, Portugal

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1. Introduction

The droplet method for TEWL calibration is a new technique [1,2], which has the advantages of gravimetric based calibration, good repeatability, and traceability. It can be used for calibrating any TEWL instruments that is capable of recording a time series of water vapour flux density readings.

In this paper, we will present a mathematical modeling for calibrating both open-chamber TEWL instruments and closed chamber with a cold plate condenser TEWL instruments. We will study the effect of RH sensor position, RH sensor mis-calibration, chamber length, droplet position, and instrument response speed.

2. Theory

Figure 1 shows a schematic diagram of the TEWL calibration using droplet method.

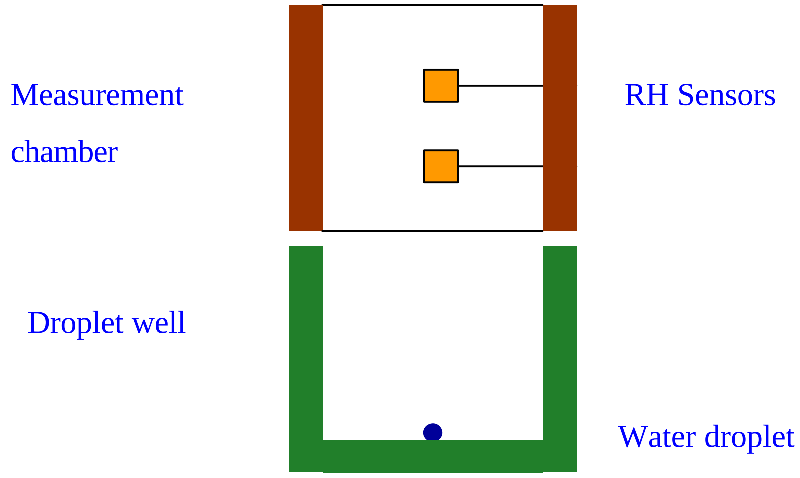
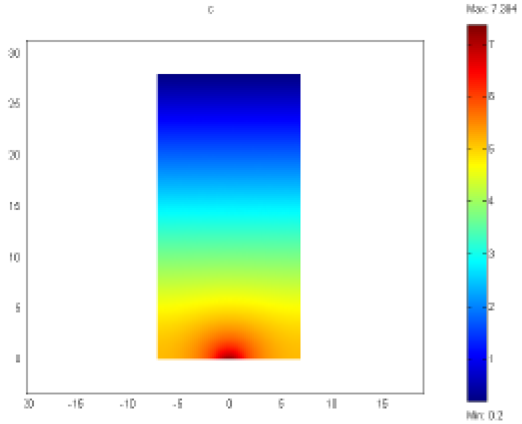


FIGURE 1. *Schematic diagram of the droplet method used to calibrate a TEWL instrument.*

2.1 Water Vapour Density Map within the Measurement Chamber

Water vapour density distribution within the measurement chamber is not uniform during TEWL calibration using droplet method. Figure 2A shows the water vapour density map within the measurement chamber at time 200 seconds. Figure 2B show the time dependent water vapour density curve at the sensor position.

Apparently, water vapour can diffuse into the chamber and reach steady state very quickly (~20 seconds). Also, if an RH sensor is a certain distance away from the droplet, say half of chamber length, then at sensor horizontal position the water vapour density distribution will be relatively uniform, there is no difference to put the sensor(s) in the centre as in open-chamber instrument, or on the chamber cylinder wall as in closed-chamber instrument with a condenser.



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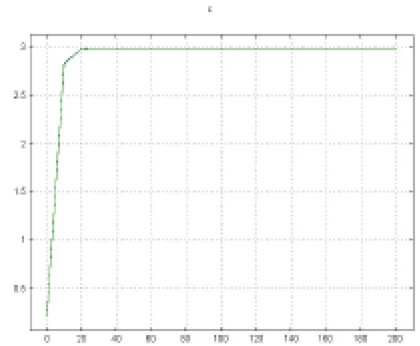

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FIGURE 2. *Finite Element simulation of water vapour density map within a TEWL measurement chamber. A: vapour density at 200sec. B: vapour density vs time (sec) at the sensor position. Water vapour diffusion coefficient of air used is 2.54×10^{-5} m^2/s.*

2.2 Effect of Sensor Position, Chamber Length & Droplet Position

There are many factors that may affect the TEWL calibration using droplet methods. Figure 3 shows the flux density map within the measurement chamber with the droplet at different positions. Figure 4 shows the flux density at the cross section of the sensor position with droplet at different positions.

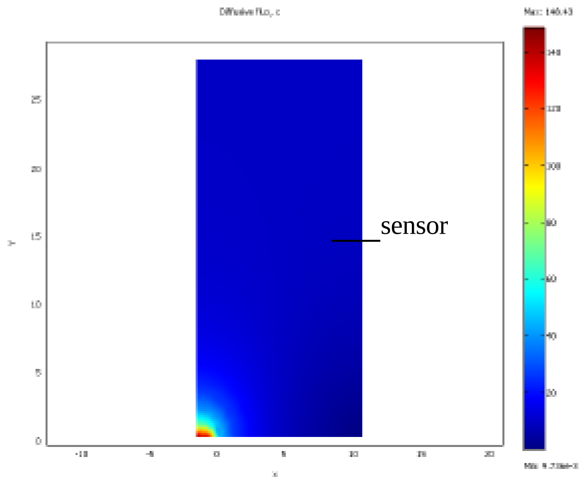

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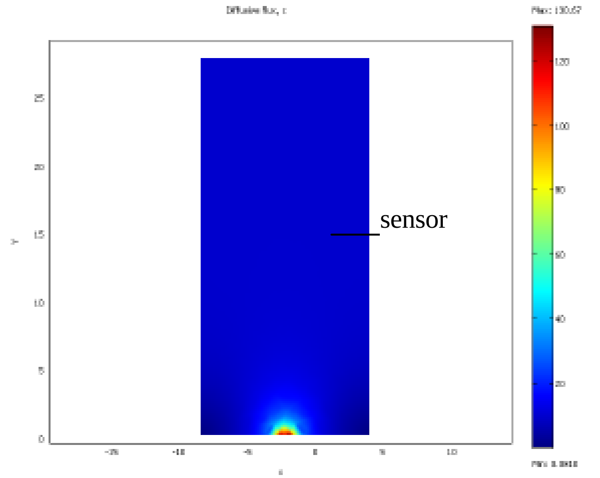
FIGURE 3. *Finite Element simulation of water vapour flux density map within a TEWL measurement chamber. A: droplet at the edge. B: droplet at the centre. C: with a longer chamber.*



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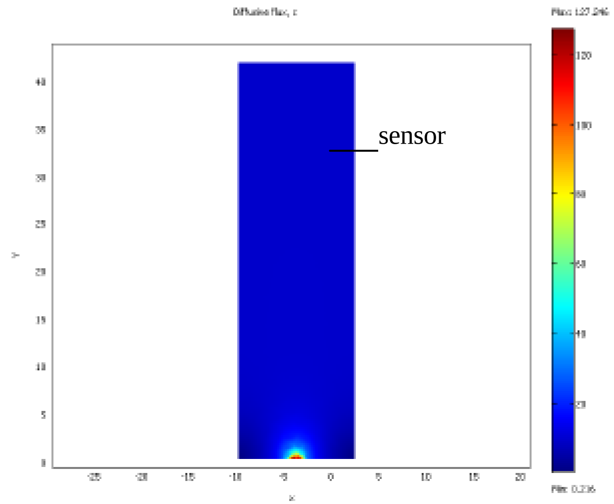
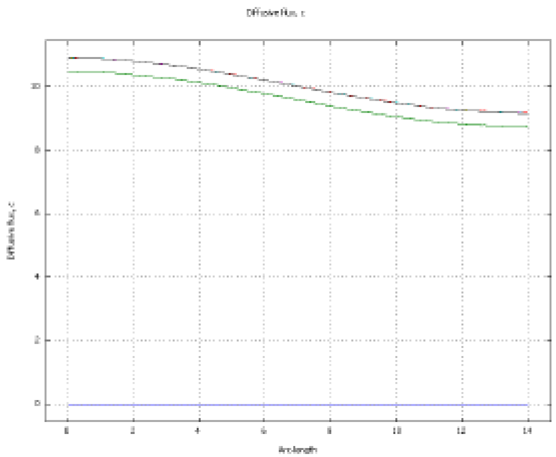


C

From Figures 3 and 4, the horizontal position of droplet will significantly affect the calibration result. However the vertical position, as long as the droplet is a certain distance away from the sensor, will not affect the calibration result. It will only make the calibration time longer.

As for sensor position, when the droplet is in the centre, it makes no difference to the calibration result, as the flux distributed evenly across the section. But when the droplet is away from the centre, then the sensor position will affect the results.



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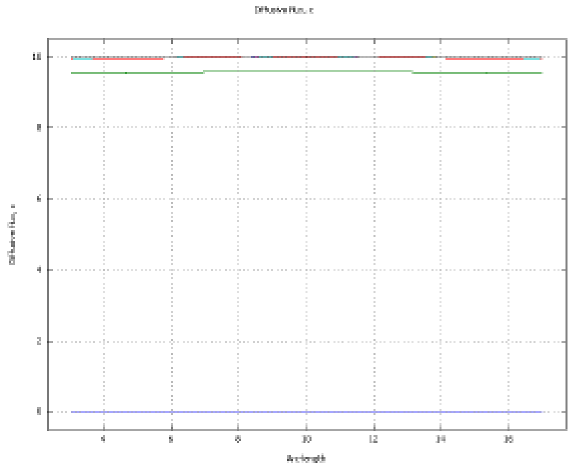

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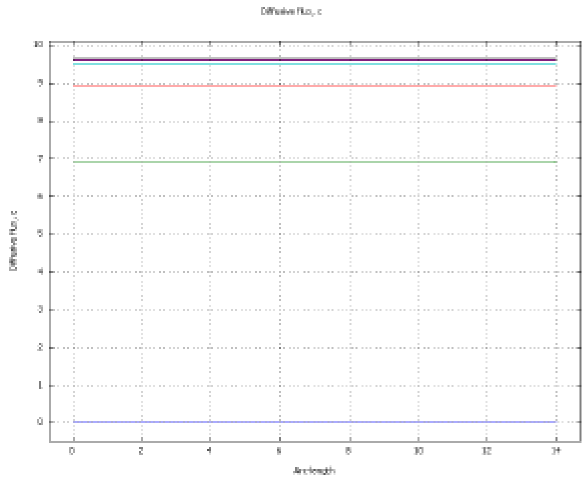

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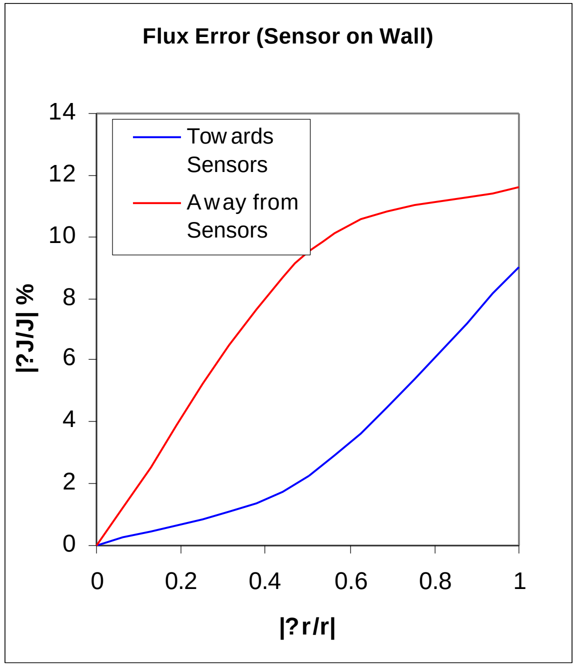
FIGURE 4. *Finite Element simulation of time dependent water vapour flux density at cross-section of sensor position. A: droplet at the edge. B: droplet at the centre. C: with a longer chamber.*

Figure 5A shows the flux error caused by droplet position when the sensor in on the wall. Figure 5B shows the flux error caused by droplet position when the sensor in the centre. When the droplet is in the centre, the sensor position will not cause any error in flux calculation.



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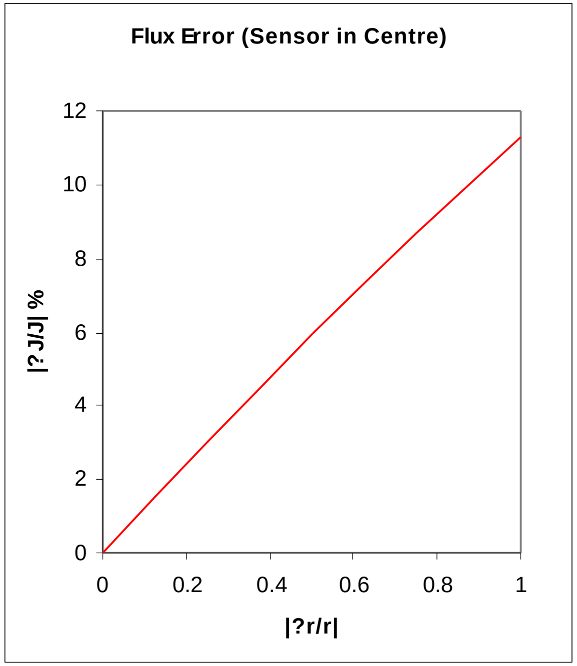

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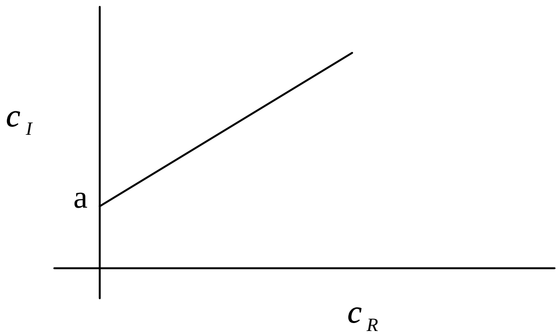
FIGURE 5. *Flux error due to droplet position calculated from the results shown in Figure 3. A: sensor on the wall. B: sensor in the centre.*

2.3 Effect Of RH Sensor Mis-calibration

The RH value measured by a sensor in a TEWL instrument (c_I) can be assumed as a function of real RH value (c_R).

$$c_I = a + bc_R$$

where a is an offset parameter due to hysteresis, and b is the slope. In an ideal world, a=0, b=1. But in practice, a$\neq$0, b$\neq$1.



Vapour density at the sensor position within the measurement chamber is given by

$$VD = c_I \cdot f(T_S)$$

where T_S is sensor temperature.

For **closed-chamber instrument with a condenser**, the flux is:

$$J = A*(VD - f(T_C))$$
$$J = A*((a + bc_R) \cdot f(T_S) - f(T_C))$$

where A is flux scaling factor, and T_C is condenser temperature.

In baseline calibration, we have

$$J = 0$$
$$A*(VD - f(T_C)) = 0$$
$$a = f(T_C)/f(T_S) - bc'_R$$

Therefore, the baseline calibration in the condenser method will correct any mis-calibration of offset parameter "a".

In flux calibration using droplet method, the integration of flux density J over time, multiplied by the cross-sectional area B can be equated to the quantity of the water droplet Q,

$$B \cdot \int_{t0}^{t1} A \cdot J \cdot dt = Q$$

$$B \cdot \int_{t0}^{t1} A \cdot b \cdot (c_R - c'_R) \cdot f(T_S) \cdot dt = Q$$

$$A \cdot b = \frac{Q}{B \cdot \int_{t0}^{t1} (c_R - c'_R) \cdot f(T_S) \cdot dt}$$

In other words, when the instrument is baseline-calibrated, the droplet calibration will correct for any mis-calibration of the sensor calibration parameter *b*.

For **open-chamber instrument with two sensors**, the flux is:

$$J = A * (VD1 - VD2)$$

$$J = A * ((a1 + b1c_{R1}) \cdot f(T_{S1}) - (a2 + b2c_{R2}) \cdot f(T_{S2}))$$

where A is flux scaling factor, VD1 is vapour density at sensor 1 and VD2 is vapour density at sensor 2. T_{S1} is sensor 1 temperature, and T_{S2} is sensor 2 temperature.

In baseline calibration, we have

$$J = 0$$

$$(VD1 - VD2) = 0$$

$$(a1 + b1c_{R1}) \cdot f(T_{S1}) = (a2 + b2c_{R2}) \cdot f(T_{S2})$$

Therefore, a baseline calibration alone is not enough to correct for any mis-calibrations of the two humidity sensors.

In flux calibration using droplet method, the integration of flux *J* over time, multiplied by the cross-sectional area *B* can be equated to the quantity of the water droplet *Q*

$$B \cdot \int_{t0}^{t1} A \cdot J \cdot dt = Q$$

$$B \cdot \int_{t0}^{t1} A * ((a1 + b1c_{R1}) \cdot f(T_{S1}) - (a2 + b2c_{R2}) \cdot f(T_{S2}))dt = Q$$

Therefore, the droplet calibration is not enough to correct any mis-calibration of the two humidity sensors. In order to get accurate flux results, in addition to the flux calibration using droplet method, the two humidity sensors also need to be calibrated separately.

2.4 Effect of Finite Instrument Response Time

All instruments have finite response time, therefore the flux measured by instrument ($J_M(t)$) equals the convolution of real flux ($J_{real}(t)$) and instrument response ($R(t)$).

$$J_M(t) = J_{real}(t) \otimes R(t)$$

During droplet flux calibration, we have

$$\begin{aligned}\int_{-\infty}^{+\infty} J_M(t)dt &= \int_{-\infty}^{+\infty} J_{real}(t) \otimes R(t)dt \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} J_{real}(t)R(t-t)dt\,dt \\ &= \int_{-\infty}^{\infty} J_{real}(t)\left[\int_{-\infty}^{\infty} R(t-t)dt\right]dt \\ &= \int_{-\infty}^{+\infty} J_{real}(t)dt\end{aligned}$$

For linear instruments, i.e.

$$\int_{-\infty}^{+\infty} R(t-t)dt = 1$$

the integration of the flux measured by the instrument ($J_M(t)$) equals the integration of real flux ($J_{real}(t)$), which means the droplet calibration will cancel out the finite instrument response time effect.

3. Conclusions

The mathematical modelling results show that:

- The water vapour can quickly diffuse into the measurement chamber and reach steady state during droplet calibration. Water vapour density and flux density distribution within the chamber is not uniform.
- Droplet position will affect the calibration result, but as long as the droplet is in the centre, and the sensor is a certain distance away from the water droplet, the radial position of the sensor, does not affect the results.
- The mis-calibration in RH sensors will also cause significant error in flux measurements. For condenser chamber method, the baseline calibration can correct the error due to hysteresis of the sensor. The droplet calibration can correct the slope parameter error, if baseline calibration has been done beforehand.
- For the open chamber method, the two humidity sensors need to be calibrated separately, before valid baseline and flux calibrations can be performed.

4. References

[1] ***Mathematical Analysis of ASTM-96 Based TEWL Calibration Method***
R E Imhof, M E P de Jesus, P Xiao and the TEWL Calibration Consortium, ISBS Conference, Orlando, Florida, USA, Oct 28-30, 2004.

[2] ***A New Calibration Method for TEWL with Traceability to Measurement Standards***
P Xiao, R E Imhof, M E P de Jesus, Y Cui and the TEWL Calibration Consortium, ISBS Conference, Orlando, Florida, USA, Oct 28-30, 2004.

5. Acknowledgements

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